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Velocity measurements close to the bed in a wave tank

By J. F. A. SLEATH
Department of Engineering, University of Cambridge

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Measurements of the velocity distribution close to the bed have been made under laminar flow conditions in a wave tank. The classical solution for the velocity distribution was found to be valid when the bed was smooth, but considerable deviations between theory and experiment were observed with beds of sand. It is suggested that these deviations were caused by vortex formation around the grains of sand. The similarity between the velocity profiles obtained in these tests and those reported by other writers under supposedly turbulent conditions suggests that even at high Reynolds numbers vortex formation may continue to be the dominant effect in oscillatory boundary layers of this sort.

1. Introduction

In 1851 Stokes presented a solution for the velocity distribution above a flat plate oscillating in its own plane in still water. Lamb (1932, p. 622) has shown that, with suitable choice of axes, this solution also describes the velocity distribution produced by waves in the laminar boundary layer over a smooth horizontal bed of negligible permeability. Various other writers have presented solutions to this problem, but it is readily shown that, to the first order in the product of wave amplitude a and wave-number k, and provided that the thickness of the boundary layer is small compared with the wavelength, all of these solutions are the same as those given by Lamb in the immediate vicinity of the bed. For $ky \ll 1$, where y is distance measured vertically upward from the bottom, the horizontal component of velocity may be written,

$$u = u_0[\cos\theta - \exp(-\beta y)\cos(\theta-\beta y)], \tag{1}$$

$$u_0 = \omega a/\sinh \kappa h, \tag{2}$$

where h is the mean water depth, ω is the angular frequency of the wave, $\theta = \omega t - kx$, $\beta = (\omega/2\nu)^{\frac{1}{2}}$ and ν is the kinematic viscosity.

These expressions have been widely used in calculations involving problems such as sediment transport and wave attenuation over beds of sand. Unfortunately the validity of the solution in such situations has not been tested experimentally. Under severe conditions the flow becomes turbulent and, as shown by Kalkanis (1964), the velocity distribution is then completely different from that indicated by (1) and (2). In addition, visual observations reported by Vincent & Ruellan (1957) and Lhermitte (1958) indicate that even when the flow remains laminar the velocity distribution over a bed of sand may be different from that over a smooth bottom. When a crystal of potassium permanganate is dropped onto the bed and becomes lodged against a protruding particle of sand, tongues of dye are seen to be thrown up from the bed at the end of each half wave-cycle. The tongues of dye are gradually carried along by the mass-transport current and consequently form one behind the other. Figure 1, which is based on photographs taken during one of the tests described below, provides an example of the resulting dye pattern. Vincent & Ruellan stated that these tongues indicate the onset of turbulence in the boundary layer, but this hypothesis cannot be correct, since in certain cases the dye pattern may preserve its identity for as much as one hundred wave periods. Another suggestion, put forward by Lhermitte and developed by Eagleson (1959), is that the tongues of dye are produced by separation of the laminar boundary layer from the bottom at a particular point in the wave cycle. However, since the point of separation moves along the bed at the speed of the wave, particles of dyed fluid would also be carried along at this speed. This is not the case and consequently the hypothesis cannot be correct. The observations made by the present writer indicate that these tongues of dye are produced by vortices formed around the grains of sand on the surface of the bed. A crystal of dye lodged against a grain colours the vortices formed by that grain. When the velocity reverses at the end of the half cycle the vortex is thrown out from the bed leaving behind a trail of dye similar to those shown in figure 1. The mixing produced by the formation and ejection of the vortices will clearly modify the velocity distribution. The object of the work described here is thus to answer three questions:

(i) Under what conditions do (1) and (2) apply?
(ii) At what point does vortex formation become important?
(iii) What is the effect of vortex formation on the velocity distribution?

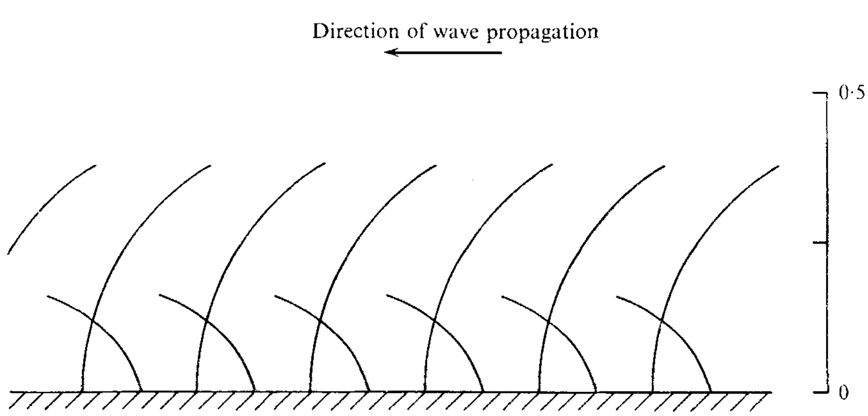

FIGURE 1. The pattern produced by tongues of dye thrown up from the bed.

2. Description of test conditions and equipment

The experiments were carried out in a wave tank 60 ft. long, 2 ft. wide and 3 ft. deep. Progressive waves were generated by a simple flap paddle oscillating at constant frequency. A filter consisting of expanded aluminium sheeting was installed 5 ft. from the wave generator, and the waves were absorbed at the far end of the tank by means of a pebble beach extending over a distance of 20 ft. In most of the experiments the slope of this beach was about 1 : 20.

Velocity measurements were made with four different beds. The case of an impermeable smooth bed was modelled by laying a sheet of plate glass on the bottom of the tank. The sheet was of length 10 ft. and was backed up at each end by steel plates, 6 ft. long, of identical thickness and with a gloss paint finish. Next, tests were carried out with two different beds of coarse sand (median diameter 0·0445 in., standard deviation 0·0183 in.). In one case the depth of the bed was approximately 13 in., and in the other it consisted of a thin layer of sand glued on to a flat impermeable bottom. Finally, measurements were made with a 13 in. deep bed of fine sand (median diameter 0·0153 in., standard deviation 0·0040 in.). The beds of sand were flat and in no test did sediment movement occur.

Measurements of the fluid velocities in oscillatory boundary layers have been made by Kalkanis (1957, 1964), Jonsson (1963) and Horikawa & Watanabe (1968). However, because of their bulk, none of the instruments used by these workers was suitable under the present test conditions. Instead, an instrument based on measurement of the tension produced in a wire by a stream flowing normal to its length has been constructed. The wire is a glass fibre of diameter 0·001 in. and length 3 in. held parallel to the bottom and at right angles to the flow. Details of the design and operation of this instrument may be found in Sleath (1969). Other equipment included hook and pointer gauges for wave-height measurement and a resistance gauge to provide a continuous check on the wave characteristics and to give a reference signal for phase measurements. Wave periods were determined using a stop-watch, and water temperatures were measured close to the beach in the region of greatest mixing.

The object of these tests was to examine the first-order component of velocity. The velocity probe responds only to the magnitude of the velocity, regardless of direction. Consequently, the velocity maxima in the positive and negative x-directions show as separate peaks on the record, and the points of zero velocity as the troughs. The maximum velocity in each direction can thus be determined separately. In the tests described below, the component of velocity in the vertical direction was always negligible compared with the horizontal component. Consequently, the average of the maximum velocities in the two directions gave the horizontal component of the first-order velocity, subject to an error of less than 1 %. A similar approach allowed the phase of the first-order component of velocity to be determined to a precision of better than 0·5°, under the test condition described below.

The test conditions were determined by the limitations of the experimental equipment and consequently covered much the same range in each of the series of tests with the four different beds. The still-water depth was usually between 11 in. and 17 in., and the wave period between 0·8 s and 5·5 s. The corresponding range of wavelengths was 34 in. up to 348 in. The wave amplitude ranged from 0·3 in. up to 1·7 in., and u_0 from 1·7 in./s up to 6·2 in./s. Further details of the test conditions are given in Sleath (1968).

3. Test results

The measurements of the amplitude $\hat{u}$ and phase of the first-order component of velocity over the glass plate are shown in figures 2 and 3. Since the conditions in the tests were very close to those assumed in the derivation of (1), it is satisfactory that the agreement between theory and experiment is so good.

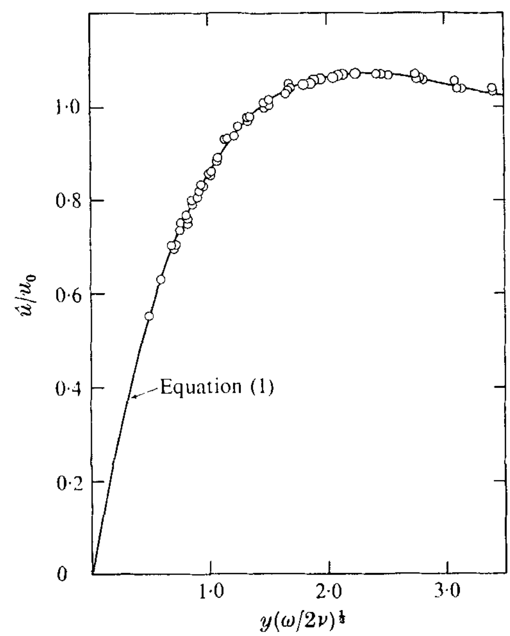

FIGURE 2. The variation in the amplitude of the first-order velocity above the glass plate.

The results of the tests with the coarse sand are shown in figures 4 and 5 for the bed of depth 13 in., and in figures 6 and 7 for the impermeable bed. Tongues of dye similar to those shown in figure 1 were observed in all of these tests. Although in some tests the velocity distribution was little different from that given by (1), in others the perturbation caused by vortex formation was very marked. It was found that in all tests the variation in amplitude was well represented by a curve of the form

$$u = u_0[\cos\theta - \exp(-\beta y/X)\cos(\theta - \beta y/X)], \tag{3}$$

where u_0 is given by (2) and X is a constant for any given test. Values of X were determined by trial and error and lay in the range $1{\cdot}0 < X < 1{\cdot}8$. For purposes of comparison, the curve corresponding to (1) has been included in figures 4 and 5, assuming a value of $X = 1{\cdot}8$ in the abscissa. The origin of the y co-ordinate in figures 4–7 has been chosen so as to give the best fit between the amplitude measurements and (3). No simple relationship could be found between the distance of this origin below the grain tips and, say, the grain diameter. Although the determination of X for a given test is thus somewhat uncertain, the deviation of the test results from the curve given by (1) is clearly very marked. The phase measurements agree quite well with (3) for values of y greater than X/β, but a very marked change is apparent close to the bed. Instead of continuing to increase, as the distance to the bed decreases, the phase of the maximum velocity with respect to that in the free stream suddenly starts to diminish. Figures 5 and 7 show, for each bed of coarse sand, the phase measurements for three tests in

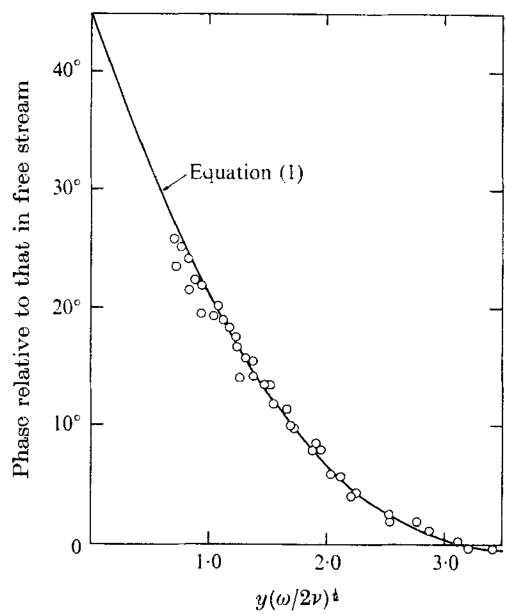

FIGURE 3. The variation in phase of the first-order velocity above the glass plate.

which this effect was particularly marked. Comparison of figures 4 and 6 and figures 5 and 7 indicates that bottom permeability has little effect on the velocity distribution. It may be shown (Sleath 1968) that the magnitude of the perturbation to the first-order velocity, introduced by a percolation velocity of amplitude V_0 into and out of the bed, is only of the order of $V_0^2/4\omega\nu X^2$. This parameter did not exceed 0·02 in any test.

Finally, the amplitude and phase measurements for three of the tests with the bed of fine sand are shown in figures 8 and 9. At distances greater than $1/\beta$ from the bed the velocity distribution is little different from that given by equation (1), whereas close to the bed the experimental points deviate markedly from the curve. It is thus not possible to reduce these results to a single curve merely by adopting a constant scaling factor X. It will be noted that the magnitude of the perturbation appears to show a strong correlation with βD. It was not possible to observe tongues of dye being thrown up from this bed of fine sand, but this is probably because the vortices formed around the grains were too small to penetrate far enough from the bed to be distinguished. The stability of the trails left by dye crystals as they fell towards the bed rules out the possibility of turbulence.

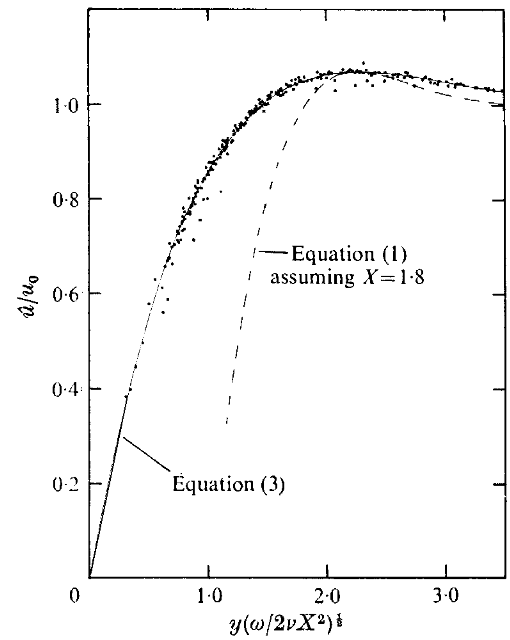

FIGURE 4. The variation in amplitude of the first-order velocity for all of the tests with the bed of coarse sand of depth 13 in.

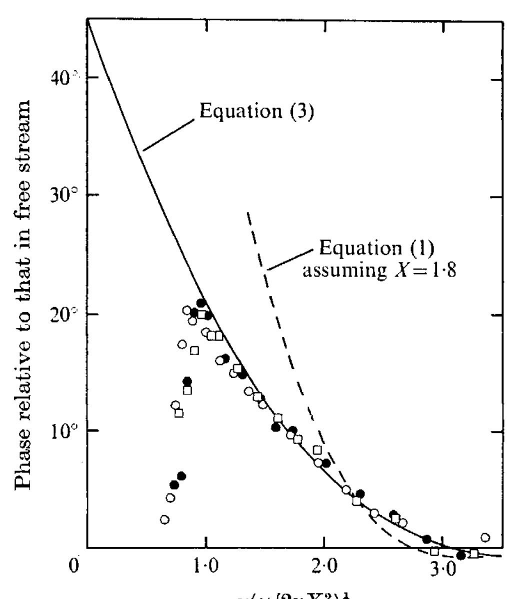

FIGURE 5. The variation in phase of the first-order velocity for three tests with the bed of coarse sand of depth 13 in.

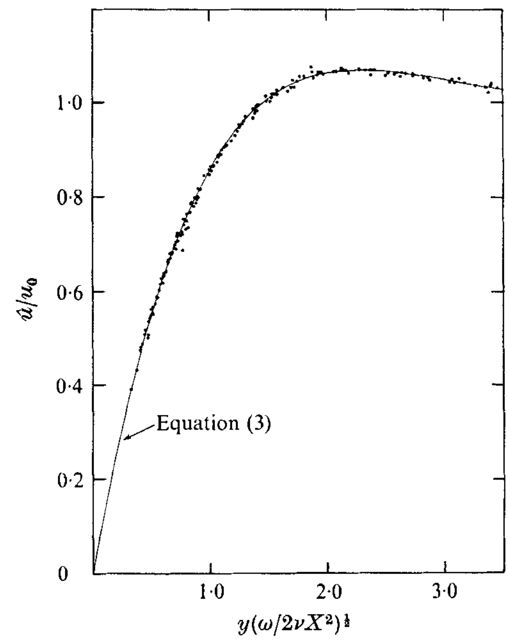

FIGURE 6. The variation in amplitude of the first-order velocity for all of the tests with the impermeable bed of coarse sand.

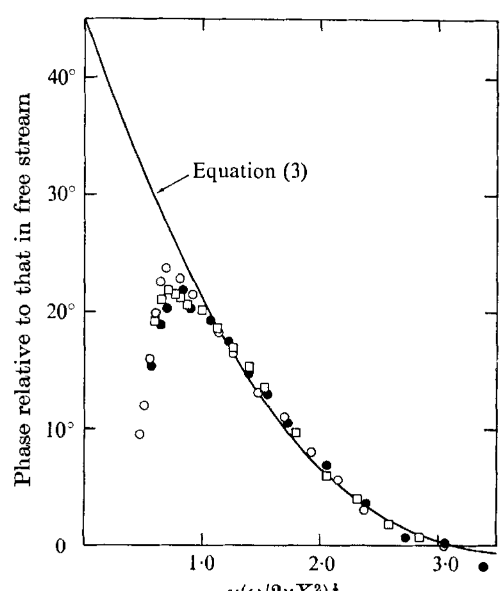

FIGURE 7. The variation in phase of the first-order velocity for three tests with the impermeable bed of coarse sand.

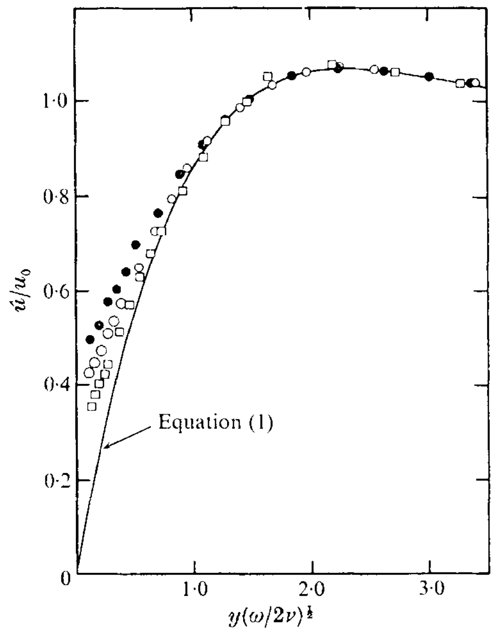

FIGURE 8. The variation in amplitude of the first-order velocity above the bed of fine sand. □, βD = 0·28; ○, βD = 0·44; ●, βD = 0·59.

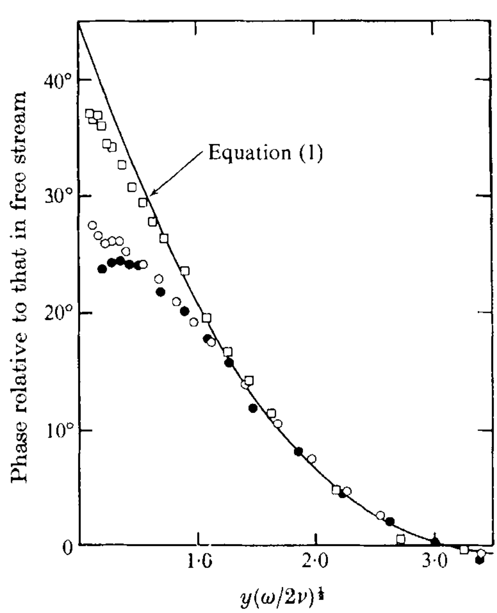

FIGURE 9. The variation in phase of the first-order velocity above the bed of fine sand. □, βD = 0·28; ○, βD = 0·44; ●, βD = 0·59.

The value of u_0 used to non-dimensionalize the amplitude measurements in figures 2, 4, 6 and 8 is the measured value. In all the 44 tests, except for the 4 tests with the glass plate where inaccuracies in wave-height measurement were detected, this value of u_0 did not differ by more than 11 % from the theoretical value given by (2).

4. Discussion

We have seen that the principal effects of vortex formation are to increase the boundary-layer thickness and to produce a rapid reduction in the phase lead of the velocity in the immediate vicinity of the bed. The increase in boundary-layer thickness is consistent with what we should expect on the basis of momentum-exchange arguments. Since vortex formation is associated with individual grains of sand, the scale of the mixing will be of the same order as the grain size. The median diameter of the coarse sand was greater than $1/\beta$ in all tests and consequently we should expect the mixing to be approximately constant across the boundary layer, whereas the grain-size of the fine sand was considerably less than $1/\beta$, so that we should expect the mixing to be more intense close to the bed. However, the exact nature of the flow is undoubtedly very complicated and no useful purpose would be served by introducing a necessarily simple mathematical model at this stage. In order to show that the rapid change in phase close to the bed is also consistent with what we should expect, let us suppose that the bed is the plane $y = 0$ and that, at any rate very close to the bed, the effect of the vortices around individual grains may be represented by a vortex sheet at $y = 0$. Before the vortices begin to form, the velocity of the fluid at $y = 0$ is zero. The result of vortex formation is thus that the velocity of the fluid immediately above is greater than it would otherwise have been. As the strength of the vortices grows, the magnitude of the perturbation to the velocity also increases. However, at the end of the half-cycle the vortices are forced out from the bed, and consequently the perturbation velocity falls sharply to zero. The process is then repeated in the opposite direction. Let us assume that the perturbation velocity at $y = 0$ during a wave cycle may be represented by:

$$\begin{aligned}\text{perturbation velocity} &= u_p(\theta+\phi)/\pi \quad (0 < \theta+\phi < \pi),\\ &= -u_p(\theta+\phi-\pi)/\pi \quad (\pi < \theta+\phi < 2\pi),\end{aligned}$$

where u_p and ϕ are arbitrary constants. If this perturbation is decomposed into a Fourier series and the unperturbed flow is given by (3), the velocity distribution in the vicinity of the bed takes the form

$$\begin{aligned}u = u_0[\cos\theta - \exp(-\beta y/X)\cos(\theta-\beta y/X)]\\ + 2u_p \sum_{n=0}^{\infty} \exp(-\beta_n y)\,[\sin((2n+1)(\theta+\phi)-\beta_n y)\\ - 2\cos((2n+1)(\theta+\phi)-\beta_n y)/(2n+1)\pi]/(2n+1)\pi,\end{aligned} \tag{4}$$

where $\beta_n = \beta(2n+1)^{\frac{1}{2}}/X$.

Figure 10 shows a number of examples of the way in which the phase varies with distance from the bed for $u_p = \frac{1}{4}u_0$ and various values of ϕ. Since at $y = 0$ the phase of the unperturbed velocity leads that in the free stream by $\frac{1}{4}\pi$, $\phi = \frac{3}{4}\pi$ might seem to be most appropriate. Smaller values of ϕ might be justified on the grounds that vortex formation does not start until the velocity has reached a certain critical value and that ejection may be delayed beyond the end of the half-cycle. However, too much stress should not be placed upon the exact values of ϕ or u_p, or upon the assumed form of the perturbation at the bed. Clearly, similar curves will always be obtained if the perturbation velocity has the asymmetrical form described above.

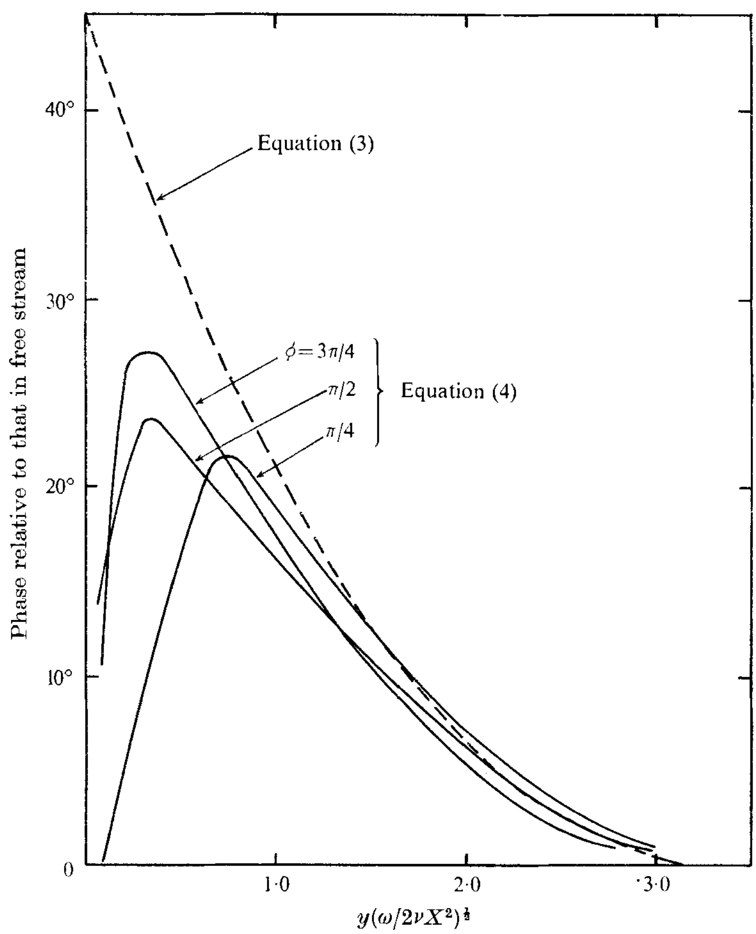

FIGURE 10. The effect of vortex formation on the variation of the phase near the bed.

5. Comparison with other work

In all the tests with the beds of coarse sand, tongues of dye were observed to remain intact right down to the bed over times large compared with the wave period. Consequently the flow cannot have been turbulent. Nevertheless, the nature of the perturbation to the velocity distribution is similar to that observed by Kalkanis (1957, 1964) with supposedly turbulent boundary layers. Relative to axes fixed in the bed, Kalkanis' velocity measurements over beds of sand also appear to follow (3), provided that the origin is chosen appropriately. Figure 11

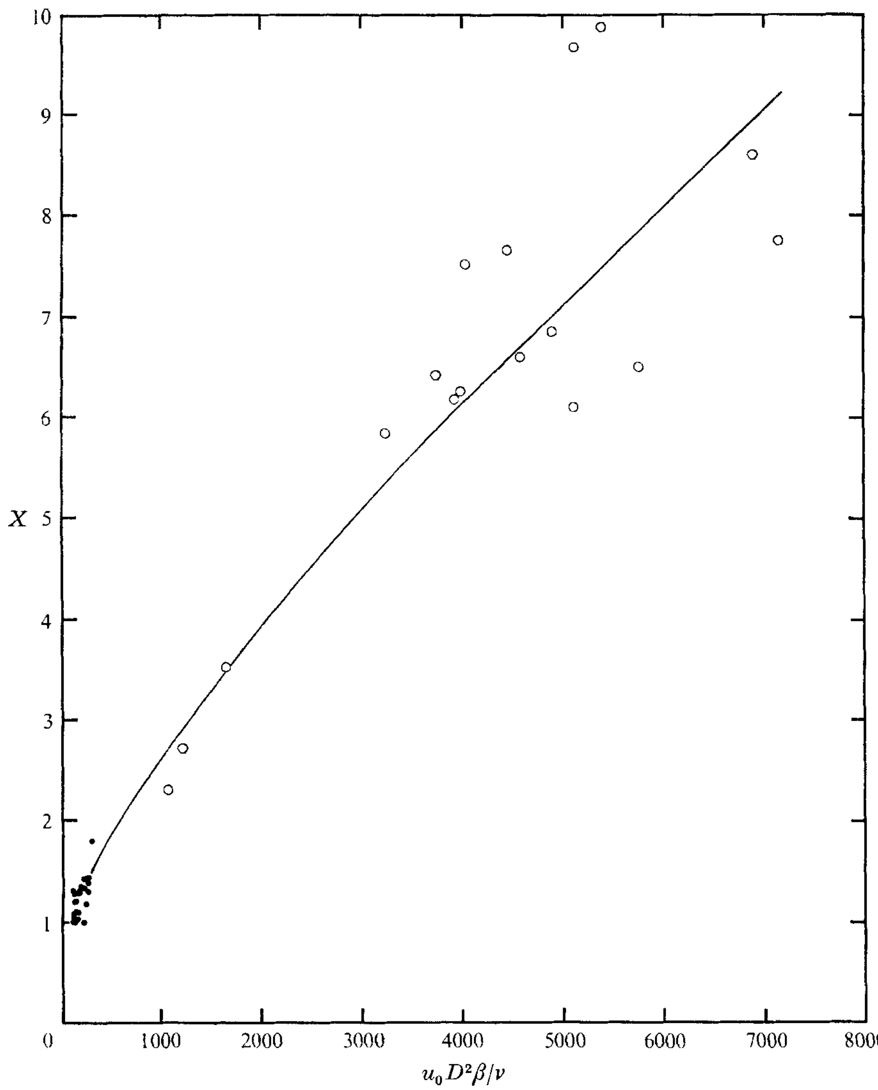

FIGURE 11. The variation of X with $u_0D^2\beta/\nu$. ●, present work; ○, Kalkanis' tests with beds of sand.

shows the way in which the parameter X varies with $u_0D^2\beta/\nu$ for Kalkanis' tests with beds of sand up to a value of $u_0D^2\beta/\nu = 8000$ and for the tests with the beds of coarse sand described above. Although the two sets of tests cover very different ranges, figure 11 does not indicate any change in behaviour of the sort associated with sudden transition from laminar to turbulent conditions. It may be that, even if turbulence does occur under more severe conditions, the flow continues to be dominated by vortex formation.

From figure 11, the critical value at which vortex formation begins to perturb the velocity distribution is $u_0 D^2\beta/\nu = 115$. Since the variation in βD is very slight in these tests, X could equally well have been plotted against $u_0 D/\nu$. In terms of this latter parameter the critical condition appears to be approximately $u_0 D/\nu = 50$. This may be compared with the value of $u_0 D/\nu = 200$ obtained from the formula proposed by Manohar (1955) for the transition from laminar to turbulent conditions with sand of median diameter = 0·0445 in. It is probable that the phenomenon observed by Manohar and his colleagues was vortex formation rather than turbulence.

The curve shown in figure 11 has been fitted by eye and corresponds to

$$X-1 = 0{\cdot}00815\left(\frac{u_0 D^2\beta}{\nu}-115\right)^{0\cdot78}. \tag{5}$$

Kalkanis' phase measurements also seem to behave in the same way as those for the beds of coarse sand described above. However, none of his measurements was made closer than $1{\cdot}5(2\nu/\omega)^{\frac{1}{2}}$ from the bed so that it is not possible to say whether there was a sharp change in phase in the immediate vicinity of the bed of the sort shown in figures 5 and 7.

Finally, it may be remarked that vortex formation and ejection is also found in flow over rippled beds. It is thus possible that the flow is dominated by the mixing produced by the vortices in this case also. The measurements of Kalkanis (1964) with two-dimensional roughness elements can be expressed by a relationship similar to (3). However, the pattern of vortex filaments in this two-dimensional flow is very different from that with beds of sand, and consequently detailed comparison of the present test results with those obtained over rippled beds would not appear to be justified.

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